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## Chapter 9

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### Interest Rates and Bond Valuation

#### My Word is My Bond...

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Loaning small amounts of money to your friends might take no more than a word or a handshake, but when dealing with large amounts of money with complete strangers, we tend to require a bit more formality. Banks are the first place we tend to consider when we think about these more formal loans, but there are other avenues for governments, municipalities, corporations, and other borrowers to raise funds, as we discussed in [Chapter 8 "Securities Markets"](#).

## 9.1 Bonds and Interest Rates

PLEASE NOTE: This book is currently in draft form; material is not final.

### LEARNING OBJECTIVES

1. Explain what a bond is and who issues bonds.
2. List and describe the key features of a bond.
3. Calculate the coupon rate given the annual coupon payments and the par value of a bond.

In this chapter, we will focus on bond instruments, which are one way to **securitize**<sup>1</sup> (to make a series of cash flows into a tradable instrument) a loan. A **bond**<sup>2</sup> is a loan packaged as a tradable security, typically with a periodic interest payment and a repayment of principal at maturity.

### Who Issues Bonds?

There are many different entities that will issue bonds, and often bonds will behave differently based upon the type of issuer. At an investment bank, bond traders will often specialize in trading one type of bond. Here are some of the more common:

- Corporation = Corporate Bonds or “Corporates”
- US Government = Government Bonds or “Govies”
- Smaller Government Entities (e.g. Cities or States) = Municipal Bonds or “Munies”
- Other Governments = Sovereign Bonds or “Sovereigns”

1. To make a series of cash flows into a tradable instrument.
2. A loan packaged as a tradable security, typically with a periodic interest payment and a repayment of principal at maturity.

Bonds originally were pieces of paper, stating all of the necessary information in the center with coupons (yes, just like the Sunday paper!) around the edge. The owner of the bond would cut off the coupon and turn it in to receive each periodic

interest payment. The terminology stuck, so even with today's electronic bonds, the periodic payments are called **coupon payments**<sup>3</sup>. Most bonds pay these coupon payments twice a year (semiannually), though once a year (annual) payments are not uncommon.

These coupon payments typically occur until the **maturity date**<sup>4</sup>, at which time the final coupon payment and the principal of the bond would be due. This principal amount is called the **par value**<sup>5</sup>. Currently, the most common par value for corporate bonds in the USA is \$1,000. When describing a bond, we typically list the **coupon rate**<sup>6</sup>, which is the annual total of the coupon payments expressed as a percentage of the par value.

For example, a bond which pays \$50 semiannually would be paying a total of \$100 ( $\$50 \times 2$ ) every year. If the par value of the bond is \$1,000, then we say that the bond has a coupon rate of  $\$100 / \$1,000 = 10\%$ .

*Equation 9.1 Coupon Rates*

$$\text{Annual Coupon Payments} \div \text{Par Value} = \text{Coupon Rate}$$

$$\text{Coupon Rate} \times \text{Par Value} = \text{Annual Coupon Payments}$$

### Coupon Rate vs. Interest Rate

It is very easy to confuse the coupon rate with the interest rate. The key item to remember is that the interest rate can change over time (as the price of the bond fluctuates), but the coupon rate for most bonds is established at the time of issue and typically doesn't change over the life of the bond. To make things more confusing, companies typically like to issue the bonds with the coupon rate being close to the prevailing interest rate! But just remember that, once the coupon rate is set at the time of issue, it doesn't change.

There do exist bonds that having "floating" coupon rates which can change, but they are the exception and not the rule. In this book, we will always assume that bonds are standard bonds with fixed coupons.

All bonds are governed by a **bond indenture**<sup>7</sup>, which is the contract which stipulates all of the specific details of the bond, including conditions for repayment

3. The periodic payments of a bond, typically paid semiannually.
4. The date on which the final coupon payment and the principal of the bond is due.
5. The principal amount of the bond.
6. The annual total of the coupon payments expressed as a percentage of the par value.
7. The contract which stipulates all of the specific details of a bond, including conditions for repayment and default.

and default, when bondholders can usually seek legal recourse for greater control of the company. Specific items in the bond indenture are referred to as **covenants**<sup>8</sup>.

### KEY TAKEAWAYS

- Bonds make coupon payments until the maturity date, at which point the final coupon payment and the par value are due.
- The coupon rate is the annual coupon payments of the bond divided by the par value of the bond.
- Coupon rates don't change over the life of the bond for almost all bonds.

### EXERCISES

1. List the key features of a bond and what they represent.
2. A typical bond is issued with a 5% coupon. If interest rates change to 6%, what happens to the coupon for this typical bond?

8. Specific items in a bond indenture.

## 9.2 Credit Risk

PLEASE NOTE: This book is currently in draft form; material is not final.

### LEARNING OBJECTIVES

1. Describe credit risk.
2. Explain why US government securities are used as a proxy for a risk-free asset.
3. Explain how ratings agencies help to identify credit risk.

Not all bonds are created equal. For example, which seems like the safer investment: loan \$1,000 to the US government, or loan \$1,000 to a small company that hasn't paid interest to its current investors for over a year? Investing in the world's largest economy probably seems like the better bet, all other things being equal. But what if the small company offered you three or four times the return on your investment? Now the decision is not so straightforward....

The major difference between our government bond and the small company's bond is our expectation of the borrower's creditworthiness. We label the uncertainty in future cash flows due to possibility that the borrower will not pay **credit risk**<sup>9</sup>. The most common credit risk is **default risk**<sup>10</sup>, which in the extreme occurs when the company doesn't have the cash to pay the necessary scheduled payment (or chooses not to). This is not the only reason; bonds can be determined to be in default for breaching specific covenants (for example, a covenant specifying that certain liquidity ratios be maintained), even if all payments have been made.

9. The uncertainty in future cash flows due to possibility that the borrower will not pay.

10. The most common credit risk, which in the extreme occurs when the company doesn't have the cash to pay the necessary scheduled payment (or chooses not to).

Since, as of the time of this writing, the US government is the largest government on Earth with the largest potential revenue stream, it has been considered the borrower with the least amount of credit risk. While not truly without credit risk (in 1979, for instance, the US government was late in a small portion of its debt payments, due to a supposed paper error!), as a proxy, we consider US government bonds to be the "risk-free" asset, in terms of credit risk.

Sometimes, credit risk can be reduced through the use of **collateral**<sup>11</sup>, or assets pledged to secure repayment. The most familiar collateral arrangement is the standard home mortgage, where the property itself is collateral against the amount borrowed. Corporations can also pledge certain assets when they choose to issue bonds, with the anticipation that the lower credit risk will correspond to a lower cost of borrowing.

To aid investors in their assessment of the credit risk of bond issuers, there exist ratings agencies which attempt to qualify the creditworthiness of each bond issue. Some of the larger agencies include: Standard and Poor’s (S&P), Moody’s, and Fitch. Ratings range from AAA (or Aaa, as different agencies have slightly different scales) for the most creditworthy debt, to D for debt that is already in default. Bonds rated at least BBB- (or Baa3) are considered “investment grade”, which can be an important distinction for many mutual funds that invest in bonds to allow the bonds to be held in their portfolios. Any bonds rated lower (BB+/Ba1 or below) are considered not investment grade, or “speculative”. These bonds are sometimes called “junk bonds” as well (though it would be important to remember that even “junk bonds” have a higher claim on a company’s assets than any shares of stock).

Table 9.1 Bond Ratings

	Moody’s	S&P	Fitch
<b>High Grade</b>	Aaa to Aa3	AAA to AA-	AAA to AA-
<b>Medium Grade</b>	A1 to Baa3	A+ to BBB-	A+ to BBB-
<b>Speculative</b>	Ba1 to B3	BB+ to B-	BB+ to B-
<b>Very Risky or In Default</b>	Caa1 to C	CCC+ to D	CCC to D

There have been some concerns over the accuracy of the ratings provided by these providers, especially after the credit crisis of 2008. Some highly rated securities ended up having default rates much higher than would have been expected, given their assigned ratings. One argument is that these securities were so complex that the ratings agencies couldn’t accurately evaluate them. Another claim was that, since ratings agencies earn their revenues by charging for assigning a rating to a security’s issue, there is a potential conflict of interest that encourages the agencies to give higher ratings than they otherwise should. Regardless of the cause, confidence in ratings agencies has been shaken, and, while the information provided by such companies is undoubtedly useful, an investor is wise to not blindly take ratings as absolute truth, especially when more complex instruments are involved.

11. Assets pledged to secure repayment.

### KEY TAKEAWAYS

- Credit risk should be a major consideration when evaluating bonds.
- US Government securities are used as a proxy for the risk-free rate.
- Ratings agencies help to evaluate credit risk, but should not be taken as absolute truth.

### EXERCISES

1. When can a bond be determined to be in default?
2. Which has more credit risk: investment grade debt or speculative debt?

## 9.3 Bond Yield

PLEASE NOTE: This book is currently in draft form; material is not final.

### LEARNING OBJECTIVES

1. Explain the relationship between the real interest rate, the nominal interest rate, and inflation.
2. Calculate a nominal interest rate, given the real interest rate and an inflation rate, using the Fisher equation and the more common approximation.
3. Describe the relationship between bond prices and yields.
4. Calculate the risk premium of an asset, given the risk free return and the expected return of the asset.

All investments have a certain amount of risk (even stuffing dollars under your mattress is vulnerable to house fire!), and, in a perfect world, we would be able to completely assess the risk of each investment. There are many employees, companies, and consultants whose entire focus is on evaluating risk. Arguably, all of finance boils down to the study of risk and return.

The value of a bond, like any series of future cash flows, is intrinsically related to the perceived opportunity cost of the investment, as we discussed in [Chapter 6 "Time Value of Money: One Cash Flow"](#). This cost, called the **real interest rate**<sup>12</sup>, is determined by the market by causing the price of the bond to rise or fall. When we hear newscasters speaking of “interest rates falling”, what they are really saying is that the relevant bonds have changed in price, exposing a belief by investors that the opportunity costs of holding the bond are going down. So, as a shortcut we can talk about interest rates changing, but we should remember that, ultimately, these rates are derived from the price of the bonds! An investor buying a bond at a certain price point will yield a certain return on her investment; in bond parlance, this expected return is called the bond’s **yield**<sup>13</sup>. Specifically, **yield to maturity (YTM)**<sup>14</sup> is the annualized rate of return on a bond, assuming the bond is held to maturity and all the expected cash flows occur.

12. The interest rate that only compensates for the perceived opportunity cost of the investment.
13. A bond’s expected return.
14. The annualized rate of return on a bond, assuming the bond is held to maturity and all the expected cash flows occur.

### Bond Yields vs. Prices

A bond's yield is determined by its price, but we often speak of "investors require a yield of 5%". What we are really saying in this case is that investors will only pay a price for the bond that will give them a 5% return on their money.

If investors determine they need a higher yield, then they won't be willing to pay as much up front, and bond prices should fall. Thus, as yields rise, prices fall (and vice-versa). Bond prices and yields are inversely related!

Each bond faces a host of additional factors, all of which must be accounted for to properly assess the value of a bond. As time goes by, the purchasing power of a dollar (or whichever unit of currency we are using) could deteriorate; this we call **inflation**<sup>15</sup>. Opportunity cost and inflation factored together are typically labeled the **nominal interest rate**<sup>16</sup>.

If both the inflation rate and the real rate of interest are low (that is, < 5% each), then we can safely approximate the nominal interest rate as:

*Equation 9.2 Approximate Nominal Interest Rate*

*Real Rate of Interest + Inflation Rate  $\approx$  Nominal Rate of Interest*

$$r^* + \pi \approx r$$

We use  $r^*$  (pronounced "r-star") to represent the real rate of interest and  $\pi$  (the Greek letter "pi"), which here is **not** the circle constant of 3.14, to represent inflation. Why have finance scholars chosen to use  $\pi$ ? Because we thought using  $i$  would be too confusing, since both "interest" and "inflation" start with the same letter....

15. The deterioration of the purchasing power of a currency.

16. The interest rate that compensates for opportunity cost and inflation factored together.

This equation has the benefit of being easy to remember and can be used for fairly accurate results. While historically it has been safe to assume a real interest rate at or below 5%, there have been many occurrences of large rates of inflation much larger than 5% throughout the world. When this is the case, it is more prudent to utilize the more accurate Fisher equation for interest rates:

*Equation 9.3 Fisher Equation for Interest Rates*

$$(1 + \text{Real Rate of Interest}) \times (1 + \text{Inflation Rate}) = (1 + \text{Nominal Rate of Interest})$$

$$(1 + r^*) \times (1 + \pi) = (1 + r)$$

While a little more cumbersome to remember and use, it is the more accurate, especially if inflation is high.

This combination of the real rate of interest and the inflation rate should be the return required on an asset that is free of any other risks; we call this the risk-free rate of return and designate it:  $r_{RF}$ . As of the time of this writing, standard practice is to use US government bonds as a proxy for such a risk-free asset.

We already know of one risk, credit risk, that will cause investors to demand higher yields, but there are many other potential sources of risk. For example, if a bond isn't likely to be easily tradable, then investors could want higher yields to compensate. All of these will contribute to the investment's **risk premium**<sup>17</sup>, or amount of return required by investors over the risk-free asset.

*Equation 9.4 Risk Premium*

$$\text{Required Rate of Risk-Free Asset} + \text{Risk Premium} = \text{Required Rate of Asset}$$

$$r_{RF} + RP_A = r_A$$

$$\text{therefore: } RP_A = r_A - r_{RF}$$

**KEY TAKEAWAYS**

- Real interest rate + Inflation = Nominal Interest Rate (when inflation is reasonable).
- Bond prices go up, yields go down. Bond prices go down, yields go up.
- The risk premium of an asset is the extra amount of yield needed above the risk-free rate to compensate for the higher level of risk.

17. The amount of return required by investors over the risk-free asset.

## EXERCISES

1. If the expected rate of inflation increases, what will happen to the nominal rate of interest? What should happen to bond prices?
2. If the real interest rate is 2% and expected inflation is 3%, what is the approximate nominal interest rate? Using the Fisher equation, what is the exact nominal interest rate? Repeat with an expected inflation rate of 15%.
3. A company just issued a poor earnings report, throwing into doubt their ability to continue their debt payments. What should happen to their risk premium? What should happen to the price of their bonds?

## 9.4 Bond Valuation

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### LEARNING OBJECTIVES

1. Calculate the price of a bond.
2. Explain how bond prices are quoted.
3. Calculate the YTM of a bond.

The financial value of any asset is the present value of its future cash flows, so we already have the tools necessary to start valuing bonds. If we know the periodic coupon payments, the par value, and the maturity of the bond, then we can use our time value of money skills from [Chapter 7 "Time Value of Money: Multiple Flows"](#) to solve for either price or YTM, given the other.

For example, if I know that a bond with a 5% annual coupon has 7 years to maturity, a \$1,000 par value, and has a YTM of 6.5%, I can figure out its price. Since payments are yearly:  $n = 7$  years,  $r = 6.5\%$  per year,  $PMT = (\$1,000 \times 5\%) = \$50$  per year, and  $FV = \$1,000$ .

*Figure 9.1 Bond Timeline—Annual Coupon*

Using a financial calculator or excel, or solving by hand, we should get a  $PV = \$917.73$ .

### Quoting Bond Prices

Bond prices, by convention, are quoted as a percentage of the par value. In our above example, the result of \$917.73 would be quoted as  $917.73 / 1,000 = .91773 = 91.77\%$ . And because we know they are percentages by convention, we don't include the percent sign. So our bond quote is 91.77.

If a bond is quoted at exactly 100, we say it is trading at par (since it costs the par value). Above 100, we say the bond is trading at a premium, and below 100, it is at a discount.

If our bond paid its coupon semiannually, we need to calculate the value in terms of semiannual (six month) periods. By convention, bond yields are quoted like an APR, in that they are always equal to the rate over the period times the periods in a year. Thus, our inputs should be:  $n = 7 \times 2 = 14$  semi-years,  $r = 6.5\% / 2 = 3.25\%$  per semi-year,  $PMT = (\$1,000 \times 5\% / 2) = \$25$  every semi-year, and  $FV = \$1,000$ .

*Figure 9.2 Bond Timeline—Semiannual Coupon*

Our result is a PV of \$916.71, which is only slightly different than our annual coupon bond.

### Clean vs. Dirty

For ease, we have only used example bonds with whole years left until maturity. A bond that was traded part of the way through the year would have accrued interest due to the bond seller (since the entire coupon payment will now go to the new owner).

By convention, bond prices in US markets are quoted as “clean prices”, meaning they ignore this accrued interest. This makes it easier to compare bonds day-to-day, since the accrued interest changes every day (and resets to zero when the coupon payment is made). When a trade actually occurs, the clean price plus the accrued interest will be combined to make the actual payment (the “dirty” price).

Solving for YTM is similar. If the price of our above semiannual coupon bond rose to a quoted 94.35, what is the YTM?

*Figure 9.3 Bond Timeline—Semiannual Coupon Solving for YTM*

$n = 14$  semi-years,  $PMT = \$25$ ,  $FV = \$1,000$ , and  $PV = -(\$1,000 \times .9435) = -\$943.50$ . We represent the price as a negative number since it is a cash outflow. Solving for  $r$  gives a result of 3.00% for the semiannual period, or  $3.00\% \times 2 = 6.00\%$  for the quoted YTM.

Excel has two functions that can be used to directly solve for bond prices and yields. Both require actual calendar dates for settlement and maturity of the bond; since all we know of our bond is that it has 7 years to maturity, we'll pick dates that are 7 years apart. Both also require an input of how much of the par value is received at redemption, quoted as a % of par. Since we are receiving all of the par value at redemption, this number will be 100.

=PRICE(settlement date, maturity date, coupon rate, yield, redemption, pmts per year)

=PRICE("1/1/2010", "1/1/2017", 5%, 6.5%, 100, 2)

91.67

=YIELD(settlement date, maturity date, coupon rate, price, redemption, pmts per year)

=YIELD("1/1/2010", "1/1/2017", 5%, 94.35, 100, 2)

.0600 or 6.00%

### KEY TAKEAWAYS

- Use TVM equations or excel functions to solve for the price or yield of bonds.
- Bond prices are quoted as a percent of par, so they should be near 100. Above 100, the bond is trading at a premium, and, below 100, at a discount.

### EXERCISES

1. A bond is trading at a premium. What does that say about its YTM relative to its coupon rate?
2. A 7% annual coupon bond has a YTM of 8% and has 5 years until maturity. What is the quoted price if the par value is \$1,000? What is the quoted price if the par value is \$5,000?
3. A 6% semiannual coupon bond has 8 years until maturity. Its quoted price is 107.50. If its YTM stays constant, what should be its quoted price 1 year from now?

## 9.5 The Bigger Picture

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### LEARNING OBJECTIVES

1. Describe how bonds fit into the larger picture of finance.
2. Explain ethical considerations regarding bonds.

Bonds allow entities to borrow money easily from investors. When banks demand too much return or are unwilling to accept more risk, bonds can be the avenue to raising more capital. Corporations (or governments, municipalities, etc.) can then use the capital raised to invest in new projects or pay for ongoing projects.

As managers, we need to consider how our actions will influence our ability to issue bonds and raise capital. As investors, we need to consider bonds as an investment class.

One side benefit of bond markets is that we can witness the perceived current value of the bond as they trade. From these prices, we can deduce the return demanded by the market, and use this information to learn more about the market (for example, risk tolerance or inflation expectation) or the issuer (for example, perceived risk of default) of the bond.

### Ethical Considerations

When issuing bonds, companies have an incentive to look like safe investments, since this will lower the return demanded by the market. Some companies will make decisions based upon how their balance sheet will appear (and how they believe it will affect the rating!) for an upcoming bond issue. The legitimacy of such manipulations can be difficult to distinguish, especially if it is solely a matter of transparency.

Ratings companies are meant to be objective judges of risk, but their revenue comes from fees paid for each bond issue, raising the specter of conflict of interest. Also,

while ratings companies also include details about the risk of an issue, many investors fail to look beyond the assigned ratings, despite the potential complexity of some bonds.

Investors need to consider that buying a bond is, in effect, supporting the borrowing of the issuer. If a government is engaging in policies contrary to the conscience of the investor, or a company's revenue source is objectionable, an investor should consider avoiding the investment.

### KEY TAKEAWAYS

- Bonds allow entities to raise capital.
- Companies can try to manipulate perception to influence ratings agencies and potential cost of borrowing.
- Investors should remember that buying a bond is, at least indirectly, supporting the operations of the corporation, government, etc. that issued the bond.

### EXERCISES

1. A large government recently embarked on a campaign of silencing “dissidents”. It periodically issues bonds to fund its debt, which includes food programs for the needy and its military payroll. How might this affect an investor's decision to buy a bond?
2. If a bond is purchased on the secondary market, it isn't directly purchased from the company. Would buying such a bond still be supporting the company's funding? Why or why not?

## 9.6 End-of-Chapter Problems

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## EXERCISES

1. A 4.25% coupon bond is currently yielding 4.5%. If the par value of this bond is \$1,000, what is the annual coupon payment?
2. If comparable bonds are currently yielding 3.5%, what coupon rate should a bond have if the company wishes to issue it at par?
3. If the par value of a 3% semi-annual coupon bond is 500,000 yen, what is the payment received every six months?
4. You receive \$50 every half year from your XYZ corporate bond. It has a face value of \$1,000. What is its coupon rate?
5. Two bonds are rated B+ and BB-. Which is probably the riskier investment? Which would you expect to have the higher yield? Is it possible to predict which would have the higher coupon rate?
6. If the real rate of return is 2.5% and the expected inflation is 4%, what is the approximate real interest rate? What is the more accurate rate found using the Fisher equation?
7. If the risk free rate is 5% and a security is yielding 9%, what is the risk premium?
8. If security A is yielding 7% and security B is yielding 7.5%, which has the larger risk premium (if we are comparing to the same risk free asset)?
9. An 11% bond pays its coupon annually and has 12 years left to maturity. If its quoted price is 106.50, what is its yield to maturity?
10. An 8% bond pays its coupon annually and has 9 years left to maturity. If its current YTM is 8.16%, what is its quoted price?
11. A bond has 15 years left to maturity and pays a semi-annual coupon. If its current YTM is 7.5% and its quoted price is 82.17, what is its coupon rate?
12. A bond was issued one year ago at par with a 4% semi-annual coupon. Today, the YTM is 5%. What should be its price?
13. Bond A has an annual coupon of 6% and bond B has an annual coupon of 9%. Both have 7 years until maturity. The market demanded interest rates for these bonds moves from 4.5% to 5.5%. What are the prices for the bonds before and after the interest rate change? Which bond is more sensitive to interest rate changes (that is, which had the greater price increase as a percentage of the pre-change price)?
14. A zero coupon bond is a bond that doesn't pay periodic coupons; it only pays the face value at maturity. A \$1,000 par zero coupon bond matures in 6 years. If comparable bonds have a YTM of 7%, what should be its quoted price?