



This is “Time Value of Money: One Cash Flow”, chapter 6 from the book [Finance for Managers \(index.html\)](#) (v. 0.1).

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Chapter 6

Time Value of Money: One Cash Flow

A Dollar Today vs. A Dollar Tomorrow

PLEASE NOTE: This book is currently in draft form; material is not final.

Saving for college. Paying a mortgage. Expecting a paycheck. Each of these scenarios contain two very important factors: money and time. All other things being equal, should we prefer to have our cash now or later?

6.1 Present Value and Future Value

PLEASE NOTE: This book is currently in draft form; material is not final.

LEARNING OBJECTIVES

1. Define and explain investments and opportunity cost.
2. Define and explain the concepts of present value and future value.

“Lend me \$10 for the pizza tonight, and I’ll give you \$15 when I get my paycheck next week. Okay?” This was a common way for Mary to hit up John for some cash. Thankfully, Mary didn’t ask too often, and was always good to her word. But John only had \$50 in his wallet, just enough for the new video game that is coming out this weekend...

If John could time-travel (bear with me for a second...), he could loan the money to Mary, travel to next week to collect the \$15, travel back to today, and have enough for the video game all while making \$5. Of course, then Mary probably wouldn’t be asking to borrow John’s money at all, since she could just go forward in time to collect her paycheck. Since, as of this writing, time-travel is still an impossibility, John instead has to make a choice to commit his limited resources faced with a set of potential outcomes; he must choose which **investment**¹ is more desirable. He could deny Mary, but then he wouldn’t gain the extra \$5. But if he lends the money to Mary, he won’t have enough to purchase the video game until Mark pays him back. John therefore faces an **opportunity cost**²: the greatest outcome from the set of choices not taken when considering an investment.

Only the Greatest...

John could also use his \$50 to purchase a math textbook, new socks, or a gift for his professor. But since he values the video game the most, that is the appropriate measure of the opportunity cost.

1. A choice to commit limited resources faced with a set of potential outcomes.
2. The greatest outcome from the set of choices not taken when considering an investment.

Mary has already faced a similar decision and made her choice: she is willing to forego \$15 in a week to receive \$10 now (presumably to exchange for pizza for her growling stomach). Specifically, she has determined that the **present value (PV)**³ (the equivalent value of an outcome in today's cash) of \$15 at the time of her paycheck is less than \$10 (otherwise, she wouldn't be offering those terms).

Figure 6.1 *Comparing Outcomes*

There is no way to directly compare \$10 today and \$15 in a week, since the \$15 is a **future value (FV)**⁴. A future value is a cash flow at any time later than the present, so comparing a PV and an FV would be like comparing apples and oranges. The only proper comparison is to find the present value of each cash flow. Note that the present value of \$10 today must be equal to \$10, by definition.

We have to make similar choices every day (for example, should I put in an hour of work studying now with the hopes of eventually earning a higher grade on the exam, but foregoing that television program). Technically, we could talk about the present value of an hour of labor in the future in terms of today's hour of labor. For now, we will restrict ourselves to only considering financial investments, where the inputs and the outcomes are all quantifiable in monetary terms.

Which is worth more, a dollar today or a dollar tomorrow? (Feel free to substitute any currency.) Mary is willing to pay \$15 dollars in a week for \$10 today, so today's dollars must be worth more to her. The reason she is offering the higher future amount is, presumably, because she intends to do something with those dollars in the meantime that is of higher value than paying \$5 of interest. Could the reverse ever be true? No! A dollar today could easily be turned into a dollar tomorrow by just holding on to it and not doing anything, so no one would ever accept fewer dollars tomorrow.

3. The equivalent value of an outcome in today's cash.

4. A cash flow at any time later than the present.

Picking Nits

If there was some cost to holding dollars, say because you had so many you had to hire a guard to make sure nobody stole your cash, then it might be possible for a dollar tomorrow to be worth more. But that assumes there is nothing else for you to do with your money better than holding it (even spending it!), and that the cost of the guard is not trivial. For all practical purposes, a dollar today is going to be worth more than a dollar tomorrow.

KEY TAKEAWAYS

- Everyone's resources are limited, so committing to an investment has an opportunity cost equal to the best alternative use of the resources.
- Comparing PVs with FVs tell us nothing. Comparisons can only be made with cash flows evaluated at the same point in time.
- A dollar (or euro, or yen, etc.) today is worth more than a dollar tomorrow.

EXERCISES

1. What is the relationship between present value and future value?
2. What are the opportunity costs of attending college? In general? For your 9am class tomorrow morning?

6.2 Interest

PLEASE NOTE: This book is currently in draft form; material is not final.

LEARNING OBJECTIVES

1. Define the concepts of principal and interest.
2. Describe how an interest rate is derived from a PV and FV.
3. Describe how a market interest rate is determined.

Continuing our discussion of John and Mary: suppose John declines the original deal, and Mary counterproposes, “Could you lend me just \$8, then? I’ll still pay you the full \$15 next week.” We now know that \$8 is greater than the present value of \$15 to Mary. If John were to now accept the proposal, we would further know that the PV of \$15 to John is somewhere between \$8 and \$10, since he accepted \$8 but rejected \$10. If Mary continued to make proposals to John, she would eventually find the point where John is indifferent between giving the agreed sum now and receiving \$15 later. This point of indifference must be the present value to John of \$15 in a week. Of course, John could do the same to Mary to find her PV. As long as the PV of \$15 in a week for Mary is less than John’s PV, they should be able to come to an agreement. Of course, if they’re exactly equal, then they could agree, but both would be indifferent to doing the trade.

“Okay, I Guess....”

In everyday conversation, when we speak of being indifferent, that usually means something negative. When we say “indifferent” in economics and finance, we mean it literally: just as happy to do the trade as not. If we got one penny more, we’d always do the trade, and one penny less and we’ll say, “No thank you!”

Let's assume Mary's PV is exactly \$9. We can now quantify the opportunity cost for Mary as $(\$15 - \$9) = \$6$. Mary is willing to pay \$6 future dollars to borrow \$9 for a week. In a loan, we call the original borrowed amount (\$9) **principal**⁵ and the difference between the amount borrowed and the amount repaid (the extra \$6) **interest**⁶. If we look at the ratio of the interest to the principal, we get an **interest rate**⁷. In this case, the interest rate for the week is $(\$6 / \$9) = 66.67\%$.

Mary will, as a borrower, try to get the lowest interest rate she can find; perhaps Martha down the hall will be willing to loan her \$9 and only ask for \$14 in a week. If so, she will only pay \$5 interest, for a weekly interest rate of $(\$5 / \$9) = 55.56\%$. And John, as the lender, wants to receive the highest interest rate he can for his loan. If there are other potential borrowers, he'll pick the one willing to accept the highest rate (assuming, of course, that his PV of the principal plus interest is greater than the amount borrowed). As we add more potential lenders and borrowers to the mix, the invisible hand takes over and we should find an equilibrium interest rate, or market interest rate, emerge. Note that interest rates are implied by the market, based on the participants' relative valuations of cash at different time periods. We will often, as a shorthand, speak of the interest rate as a given, but it is good to remember that ultimately the interest rate is the derived number.

If the market is large enough, then if Mary wanted to borrow twice as much (\$18), she should be able to still apply the same interest rate to her principal to find the interest owed. So if Martha is willing to lend Mary \$18 at 55.56%, Mary will owe her: principal + interest = $\$18 + (\$18 \times 55.56\%) = \$18 + \$10 = \$28$, or twice Martha's original agreed FV of \$14. Note that if we know the interest rate for the period, r , then the relationship between the PV and the FV will be:

Equation 6.1 Interest Rate Matches Time Period

$$\text{principal} + \text{interest} = \text{FV}$$

$$\text{PV} + (\text{PV} \times r) = \text{FV}$$

$$\text{PV} \times (1 + r) = \text{FV}$$

5. In a loan, the amount originally borrowed.

6. In a loan, the difference between the amount borrowed and the amount repaid.

7. The ratio of the interest to the principal for a loan. Typically expressed as an APR.

Interest, at a fundamental level, represents the costs (especially opportunity costs) of an investment; if the market is competitive, then a competitive interest rate can be obtained from another choice of investment for the resources. Included in these costs are the perceived risks of an investment. For example, Mary has always been a reliable borrower in the past, but if John believes that there is a chance Mary might not pay him back, then he might demand a higher FV for the same PV, effectively raising the interest rate.

KEY TAKEAWAYS

- principal + interest = FV
- Interest is derived from the difference between PV and FV and is compensation for the costs (including opportunity costs and the cost of perceived risk) of an investment.
- The market interest rate is derived from the relative values of PV and FV by the participants.

EXERCISES

1. If a bank account begins the year with a \$50 balance, but ends the year with \$55, what is the implied annual interest rate?
2. If interest rates are currently 5% per year, than what is the interest earned on a \$200 investment in one year? What is the total FV?
3. If at the end of the year an investment is worth \$480, and the implied interest rate is 20% per year, what was the PV of the investment at the beginning of the year?

6.3 Simple Interest

PLEASE NOTE: This book is currently in draft form; material is not final.

LEARNING OBJECTIVES

1. Define simple interest.
2. Calculate simple interest for an investment.

What do you think should happen if Mary wishes to borrow the \$9 from Martha for two weeks? Should she assume she'll only have to pay the same \$5 of interest? Of course not! It is likely to expect that Martha will desire more interest to compensate her for the opportunity cost of the second week. So it is always important to note for what period an interest rate applies! For example, it's no use using a 2-year interest rate if you need a 1-week rate!

Let's assume that Martha is willing to lend to Mary for the second week for an additional \$5; determining interest in each period from outstanding principal only is known as **simple interest**⁸. Two weeks of borrowing means $(2 \times \$5) = \10 of interest owed, for a final payment of $(\text{principal} + \text{interest}) = (\$9 + \$10) = \19 .

Equation 6.2 Simple Interest for 2 Periods

principal + first week's interest + second week's interest = FV

$$PV + (PV \times r) + (PV \times r) = FV$$

$$PV \times (1 + r + r) = FV$$

$$PV \times (1 + 2r) = FV$$

The total interest paid is \$10 vs. \$9 borrowed, so the implied 2-week interest rate is $(\$10/\$9) = 111.11\%$. Note that this is precisely twice the 1-week rate of 55.56%. To generalize for more periods, since the interest paid will be the same for each period, we can simply multiply the interest rate by the number of periods, n , to get the effective rate for that period.

8. Determining interest in each period from outstanding principal only.

Equation 6.3 Simple Interest for n Periods

$$PV \times (1 + nr) = FV$$

KEY TAKEAWAY

- Simple Interest: $PV \times (1 + nr) = FV$

EXERCISES

1. If \$300 is invested at a 4% interest rate, how much simple interest will be earned in 4 years? What is the total FV of the investment?
2. If a loan shark offers to give you \$5,000 today in return for \$8,000 in three weeks, what is the implied weekly interest rate assuming simple interest?
3. A bank account currently has a balance of \$720 dollars. If the initial deposit was made four years ago, and the simple interest rate is 5%, what was the initial deposit?

6.4 Compound Interest

PLEASE NOTE: This book is currently in draft form; material is not final.

LEARNING OBJECTIVES

1. Define compound interest.
2. Calculate compound interest for an investment.

Now what happens if Martha takes the following line of thought: “Mary owes \$14 at the end of week 1, so the second week is as if she is borrowing \$14 to pay her first week’s debt. I should be charging her 55.56% of \$14, or \$7.78, interest for the second week!”

Where does the extra \$2.78 of interest (\$7.78–\$5) come from? Martha is now assuming that in the second week, additional interest needs to be paid on the first week’s interest of \$5. Since 55.56% of \$5 is \$2.78, Mary would owe even more interest for each week she doesn’t repay the loan, including interest-on-interest, interest-on-interest-on-interest, etc. This state of affairs, where interest is determined in each period based upon principal and accrued interest is called **compound interest**⁹.

Compound interest is much more common than simple interest; unless specifically stated otherwise, it is best to assume interest will be compounded. Compounding also introduces another complexity, the idea of a **compounding period**¹⁰: how frequently interest-on-interest is determined. In this case, our compounding period is weekly, so we need to use the weekly rate of interest, and charge interest-on-interest appropriately.

Equation 6.4 Compound Interest for 2 Periods

principal + first week’s interest + second week’s interest + interest-on-interest = FV

$$PV + (PV \times r) + (PV \times r) + [(PV \times r) \times r] = FV$$

$$PV \times (1 + r + r + r^2) = FV$$

9. Determining interest in each period based upon principal and accrued interest.

10. How frequently interest-on-interest is determined.

$$PV \times (1 + r)^2 = FV$$

Equation 6.5 Compound Interest for 2 Periods (alternative)

amount owed at week 1 + interest on amount owed at week 1 = FV

$$[PV + (PV \times r)] + [PV + (PV \times r)] \times r = FV$$

$$[PV \times (1 + r)] + [PV \times (1 + r)] \times r = FV$$

$$[PV \times (1 + r)] \times (1 + r) = FV$$

$$PV \times (1 + r)^2 = FV$$

Considering that the total owed at period n must be $(1 + r)$ times what is owed the prior period gives us the generalized form for compound interest:

Equation 6.6 Compound Interest for n Periods

$$PV \times (1 + r)^n = FV$$

Geometric Growth

Compound Interest for n Periods is one of the most important equations in all of finance and, actually, many of the sciences as well. Anything that grows at a constant rate based on the amount the period before (for example, bacteria in a petri dish) can be modeled by this equation.

Now that we have a generalized equation, we can rearrange it to solve for more than just FV. This equation is so important that most financial calculators and spreadsheet programs have dedicated functions that can be used to solve for any of the variables given the other three.

KEY TAKEAWAYS

- $PV \times (1 + r)^n = FV$
- Compound interest includes interest-on-interest.

EXERCISE

1. If \$300 is invested at a 4% interest rate, how much compound interest will be earned in 4 years? How much more compound interest is earned vs. simple interest (in other words, what was the interest on interest)? What is the total FV of the investment?

6.5 Solving Compound Interest Problems

PLEASE NOTE: This book is currently in draft form; material is not final.

LEARNING OBJECTIVES

1. Create timelines to solve single cash flow problems.
2. Calculate the FV of a cash flow.
3. Calculate the PV of a cash flow.
4. Calculate the interest rate implied by two cash flows.
5. Use a calculator or spreadsheet to calculate FV, PV, r , or n , given the other 3 inputs.

When solving compound interest problems, the first crucial step is to visualize the problem as a timeline. In creating the timeline, there are two important conventions to follow. First, cash inflows should be represented coming toward the reader (pointing down), whereas cash outflows are away (pointing up). Second, the PV is always earlier in time (to the left) of FV.

Figure 6.2 *Basic Timeline*

We will demonstrate how to solve these problems by using the equation and by using a financial calculator and spreadsheet software. Please note that, when using a calculator or spreadsheet, cash inflows will be represented using positive values, while outflows will be negative. Also, while the inputs are fairly standardized among brands, please see the appendix to this chapter for differences among the popular calculator and spreadsheet brands.

Solving for FV

Susan deposits \$2,000 into a savings account. If the interest rate is 2% compounded annually, what is her expected balance in five years?

Since Susan is giving her money to the bank, that is a cash outflow. In five years, she could potentially withdraw the money, causing an inflow.

Figure 6.3 *FV Example Timeline*

Note that if we solve the problem from the bank's point of view, the only difference is the direction of the cash flows. The numbers will necessarily have to be the same!

Figure 6.4 *FV Example Timeline (Bank's View)*

We can solve this problem without resorting to formulas or technology by reasoning through the balances at the end of each of the years. After the first, her \$2,000 will grow by 2%: $\$2,000 + (\$2,000 \times 2\%) = \$2,000 + \$40 = \$2,040$. For year two, the entire balance will grow by another 2%: $\$2,040 + (\$2,040 \times 2\%) = \$2,040 + \$40.80 = \$2,080.80$. And so on until we reach a year five balance of \$2,208.16.

Figure 6.5 *Ending balances*

Solving with the formula yields the result faster, but it is very important to conceptualize why the formula works! Using a $PV = \$2,000$, $n = 5$ years, $r = 2\%$ per year, results in:

$$\$2,000 \times (1 + .02)^5 = \$2,208.16$$

. Note that we must use the decimal representation of the interest rate (.02 instead of 2%) for the formula to work properly.

Sanity Check

Since the difference between simple interest and compound interest are relatively small, we can quickly use simple interest to “sanity check” our work for errors. 2% of \$2,000 is \$40, so with simple interest we would get \$40 per year, or $5 \times \$40 = \200 total interest. Compounding should yield slightly more than simple interest, which we can verify only adds \$8.16 more. Of course, the higher the interest rate or the more periods there are will cause the difference to increase, but it’s a useful benchmark to test that we are in the ballpark.

Solving with a financial calculator is fairly straightforward, once you’ve identified the proper inputs and account for a few minor differences. The first is that cash inflows are represented as positive numbers and outflows as negative numbers. Thus, to represent our problem accurately, the PV should be entered as a negative number (since Susan is depositing the money in the bank, which is a cash outflow. Of course, if we solve this problem from the perspective of the bank, with a positive PV, we will get the same answer for FV, just with the sign flipped!). The other important difference for financial calculators is that they expect the interest rate as a percentage, not decimal. For now, you can ignore the **pmt** key.

<CLR TVM> -2000 <PV> 5 <N> 2 <I> <CPT> <FV>

should show the proper solution of 2,208.16.

Using a spreadsheet function is very similar to a financial calculator. **pmt** will probably be a required input, so for now just enter 0; we’ll explain what this is used for in the next chapter.

=FV(rate, nper, pmt, pv)

=FV(2%, 5, 0, -2000)

2,208.16

Also, when determining the inputs for a problem, no matter what method you use, it is a good habit to write the time units for n and r , as they **must match the compounding period**. Traditionally, interest rates are quoted as an **Annual Percentage Rate (APR)**¹¹, or the compounding period’s interest rate multiplied by the number of periods in a year. Since the compounding period in our sample

11. A rate quote determined by multiplying the compounding period’s interest rate by the number of periods in a year.

problem was yearly, we didn't need to take this into account, but most real-world examples tend to have shorter compounding periods (semi-annually, monthly, or daily). Unless specifically stated otherwise, all subsequent interest rates in this book should be assumed to be quoted as APRs.

Equation 6.7 Periodic Interest Rate from APR

APR / # of periods in a year = periodic interest rate

How would our answer change if the interest was compounded monthly? PV would still be \$2,000, but r would need to be $2\% / (12 \text{ months per year}) = 0.1667\%$ per month, and n would need to be $(5 \text{ years} \times 12 \text{ months per year}) = 60$ months. Now that r and n both match the compounding period, we can use our equation: $\$2,000 \times (1 + .001667)^{60} = \$2,210.20$. Compounding more frequently will get Susan about \$2 more in interest; this might seem like a small difference, but for higher interest rates or longer duration, more frequent compounding can have a larger impact.

Common Formula Mistakes

Pay attention to the details when using the formula, as there are ample ways to make mistakes. The two biggest are using percentage when decimal is necessary, and not matching the compounding period. Since the numbers we are dealing with for interest rates can be small, a good rule of thumb is to keep four digits (not counting leading zeros!) of the interest rate before rounding. A 2% APR compounded daily has 0.005479% of interest per day. Converting to decimal (another potential source of error—count those decimal places!) gives $r = 0.00005479$.

Another common source of confusion can come from the labels “present” and “future”, since present day does not always line up with PV. Consider the following example:

Five years ago, Susan deposited \$2,000 into a savings account. If the interest rate is 2% compounded annually, what is her balance today?

This example is exactly equivalent with the first in this section, despite present day now being the FV! If one were to always assume “present value” must be “present

day”, the set-up to the problem would go all wrong. Drawing the timeline, however, safely puts the labels where they properly should be. This is why, by the way, we will almost exclusively use the labels PV and FV instead of the full words.

Figure 6.6 Graph of FV over Time for Different Interest Rates

Solving for PV

Susan has a \$3,312.24 balance in her savings account. If the interest rate has been 2% compounded annually, how much did she deposit five years ago when she opened the account?

Figure 6.7 PV Example Timeline

It is very important to note that, even though her current balance is 3,312.24, this is our FV, as we need to solve for the amount earlier on the timeline (which must be the PV). A rephrasing of the same problem would be:

Susan wants to have \$3,312.24 balance in her savings account in five years. If the interest rate is 2% compounded annually, how much must she deposit now?

$$PV \times (1 + .02)^5 = 3,312.24$$

Solving for PV yields the solution of \$3,000. Because solving for PV is such a frequent task in finance, it is worthwhile to rearrange the equation.

Equation 6.8 Compound Interest for n Periods (rearranged)

$$PV \times (1 + r)^n = FV$$

$$PV = \frac{FV}{(1 + r)^n}$$

Using a financial calculator:

<CLR TVM> 3312.24 <FV> 5 <N> 2 <I> <CPT> <PV>

should show the proper solution of -3,000.

Spreadsheet:

=PV(rate, nper, pmt, fv)

=PV(2%, 5, 0, 3312.24)

-3,000.00

Figure 6.8 Graph of PV over Time for Different Interest Rates

Solving for the Interest Rate

David currently has \$100 thousand and would like to double this money, through investing, by his planned retirement in 10 years. At what interest rate must he invest to achieve this goal?

Since David has \$100 thousand and wants to double his money, he must want to receive \$200 thousand in 10 years.

Figure 6.9 Rate Example Timeline

So Many Zeros....

Sometimes in finance we deal with very large sums of cash: thousands, millions, or even billions! Rather than type all of those zeros each time (and possibly leave one off in error), we can still use these formulas just fine by treating all of our cash numbers in their respective units. For example, this problem can still be solved properly with PV=100 and FV=200, since our units are now thousands of dollars. Just remember to be consistent!

$$100,000 \times (1 + r)^{10} = 200,000$$

.Solving for r (you'll need to take the 10th root) yields the solution of 0.07177 or 7.177%.

Using a financial calculator:

```
<CLR TVM> -100,000 <PV> 10 <N> 200,000 <FV> <CPT> <I>
```

should show the proper solution of 7.177%. Note that one of the cash flows must be negative and one positive, or the calculator will produce an error.

Spreadsheet:

```
=RATE(nper, pmt, pv, fv)
```

```
=RATE(10, 0, -100000, 200000)
```

```
0.07177
```

Growth Rate

Note that we can use this formula to solve for any rate of growth, for example, revenue growth or growth in the dividends of a stock!

Solving for the Number of Periods

David currently has \$100 thousand and would like to double this money, through investing, by his retirement. If the best interest rate he can find is 6%, how long will he have to wait to retire?

Figure 6.10 *Number of Periods Example Timeline*

$$100,000 \times (1 + .06)^n = 200,000$$

. Solving for n is a little tricky: we'd need to use logarithms. Most professionals will just use a financial calculator or spreadsheet for these types of problems.

Using a financial calculator:

<CLR TVM> -100,000 <PV> 6 <I> 200,000 <FV> <CPT> <N> should show the proper solution of 11.90 years. Again, one of the cash flows must be negative and one positive, or the calculator will produce an error.

Spreadsheet:

=NPER(rate, pmt, pv, fv)

=NPER(6%, 0, -100000, 200000)

11.90

Rule of 72

Finding time-to-double is so common that there is a quick trick to get the approximate answer. Simply divide 72 by the interest rate, and that's your time (in matching periods, of course). So if my business is growing at 9% a month, it will double in about $(72 / 9) = 8$ months. It also works to find the interest rate given number of periods: if I want my money to double by my retirement in 20 years, then I need a $(72 / 20) = 3.6\%$ rate.

KEY TAKEAWAYS

- Always make a timeline. PV is to the left of FV.
- When using a financial calculator or a spreadsheet function, cash inflows are positive and outflows are negative. There must always be at least one of each type if solving for r or n .

EXERCISES

1. If \$300 is invested at a 4% APR, how much interest will be earned in 10 years compounded annually? Compounded monthly? Compounded daily?
2. If a loan shark offers to give you \$5,000 today in return for \$8,000 in three weeks, what is the implied weekly interest rate? What is this rate quoted as an APR?
3. A bank account currently has a balance of \$7,400 dollars. If the initial deposit was made four years ago, and the interest rate is 5% compounded annually, what was the initial deposit? If the interest rate is instead a 5% APR compounded monthly, what was the initial deposit?

6.6 Effective Interest Rates

PLEASE NOTE: This book is currently in draft form; material is not final.

LEARNING OBJECTIVES

1. Explain the difference between an APR and an effective interest rate.
2. Calculate an effective interest rate.

If you are searching for the lowest interest rate for a loan, and one advertisement proclaims a 6% APR while another offers a 5.9% APR, should you always opt for the latter? It turns out to not be so straightforward because of the potential differences in compounding periods. If the 5.9% is compounded daily but the 6% is compounded yearly, you'll have the following:

Figure 6.11 Effect of Daily Compounding

Per dollar of principal, the 6% loan is .08 cents, or .08%, better over a year. Because of more frequent compounding, the realized rate of the second, 5.9% APR loan is actually 6.08%! This is called the **effective interest rate**¹², or the equivalent rate if the loan were to be compounded annually.

The easiest way to find the effective interest rate is to take a PV of \$1 and compare to the FV in one year, given the terms of the loan. Subtracting the \$1 principal from the loan nets the interest, which also equals the effective interest rate. Of course, if the compounding period is annual to begin with, the APR is the effective interest rate. Typically, however, compounding is more frequent, causing effective interest rates higher than the quoted APR (much to the chagrin of many credit card holders!). Most financial calculators include a function that computes the effective interest rate. For spreadsheet users, the function is:

12. The equivalent interest rate of a loan if it were to be compounded annually.

```
=EFFECT(APR, # of periods in a year)
=EFFECT(5.9%, 365)
```

0.0608

Using effective interest rates allows two or more offers to be directly compared without the need for further consideration of the compounding period.

KEY TAKEAWAY

- Comparing rates using APRs could be misleading; calculating the effective interest rate allows for direct comparison.

EXERCISE

1. Your credit card charges a 19% APR compounded daily. What is the effective interest rate?

6.7 End-of-Chapter Assessment

End-of-Chapter Assessment Head

First paragraph.

Paragraph.

Paragraph.

Last paragraph.

End-of-Chapter Assessment Head

- 1.
- 2.
- 3.
- 4.
- 5.

End-of-Chapter Assessment Head

1.

- a.
- b.
- c.
- d.
- e.

2.

- a.
- b.
- c.
- d.
- e.

3.

- a.
- b.
- c.
- d.
- e.